THE ASSURANCE NETWORKS APPLICATION FOR RISK MODELING OF ENTERPRISES’ INNOVATION ACTIVITY

Abstract. An example of modeling risk-contributing factors which introduce uncertainties into the innovative activity of enterprise has been discussed. The simulation method has been developed on the base of the classical theory of probability (probabilistic inference). In order to estimate the interdependent risk-contributing factors we use a simply connected tree-like network of assurance which allows modeling uncertain situations taking into account the stochastic relations which arise among situations. The obtained results of estimation of the network increasing vertices lead to a conclusion on high probability of the satisfactory condition of the innovative product orders portfolio.

Keywords: risk-contributing factors; assurance networks; risk modeling; probabilistic estimate; innovative products

The problem statement

High technology industries include traditionally those that provide a new quality of products, development of production technologies, transformation of the means of production, new forms of organization and management that increase ultimately the competitiveness of products, enterprises and the state [2]. The high cost and the potential commercial value of scientific research results force us to pay attention not only to the choices of research priorities and ways to implement them but also to the research and development process which begins from the stage of idea generation and ends with the stage of market support of the finished product and its demand making.

This process is accompanied by the various types of uncertainties that may cause risk situations entailing possible losses. Therefore, risk management can be considered as a comprehensive approach, one of the possible high technology enterprise’s management systems.

The traditional mathematical tool which is designed for risk modeling, evaluation and forecasting is the classical probability theory.

However, as the complexity of practical tasks which include a great number of casual events with various relationships between these events is increasing, within the framework of probability theory, the trend called as a probabilistic inference is emerged, i.e. on the base of all original information the probabilities of events or combinations of such events that interest us are determined. The analytical methods for probabilistic inference, Bayesian inference, probability trees, and others are widely used by now.

These methods have both advantages and disadvantages. Thus, for example, the probability trees used commonly in studies are also applied for modeling the uncertain situations of small dimensionality but while these methods utilizing the relations between events are not taken into account. These are the relations that may be present in the set of risk-contributing factors of innovation activity. This problem can be solved through the application of assurance networks that are the power tool for modeling the initial uncertain situations in many applications [1].

Analysis of the recent researches and publications

The theoretical ground of assurances networks (the alternative names are the Bayesian network, a causal network, a probabilistic network) are discussed in a number of works and the most important ones are [1, 3, 4, 5, 6]. The problems of the probabilistic inference in the assurance network are established as a definition of probabilities of the events that interest us using all information stored in the network.

A number of probabilistic inference methods are developed and the most important technique is the massage-passing method proposed by J. Pearl [5]. With this method all problems of the probabilistic inference can be divided into two types: calculation of priori conditional probabilities of events in all network vertices; calculation of posterior probabilities in all or
certain network vertices provided that there is an event (events) in a certain vertex (s) of the network. In both cases, the essence of the algorithm lies in calculation and passing special assessments through the network.

However, it should be noted that the method used for calculating simply-connected assurance networks which is its limitation. As to the multi-connected assurance networks application this method can be used with the prior transformation into a simply-connected network in the certain cases.

The aim of the article

The aim of the article is to construct and to calculate a simple, tree-like network assurance which can be applied for modeling and probabilistic estimation of the interdependent risk-contributing factors arising in the process of new product creating and it entering the market.

The main material statement

The network assurance is represented as a directed acyclic graph of the form of $G=(V,E,P)$, where $V$ is the vertex set, $E$ is the arc set, $P$ is the set of probabilistic estimate which characterizes the chances of all network events which will occur. Each vertex displays also information on unconditional and conditional probabilities of occurrence relating to the vertex. We can obtain the probabilistic estimation using either statistical (if we have the necessary data) or expert evaluation methods. Every event of any initial and intermediate vertices obtains the $\lambda$-estimate from each vertex-successor of the considered vertex. The $\pi$-estimate is transferred from the each event of the initial and intermediate vertices to every vertex-direct successor of the considered vertex. If each vertex of network obtains all necessary $\lambda$- and $\pi$-estimates than specific values for every event in the vertex are estimated, these specific values are called $\lambda$- and $\pi$-values. Thus, each arc associates with both $\lambda$- and $\pi$-estimates and $\lambda$- and $\pi$-values.

Summary conditional probability of occurrence in the vertex is calculated over $\lambda$- and $\pi$-values depending on all conditioning events occurring in other vertices. Described technique used for production intermediate vertex is represented in figure 1 [6].

In order to calculate necessary $\lambda$- and $\pi$-values in the intermediate vertex, the vertex $B$ should take the evaluation vector $\pi_B(A)$ of $\pi$-estimates from the vertex $A$, its direct predecessor, and vectors $\lambda_C(B)$ and $\lambda_D(B)$ of $\lambda$-estimates from their direct successors, vertices $C$ and $D$. Analyzing fig. 1 we have concluded that any initial or intermediate vertices take $\lambda$-estimate from all their direct successors. Any intermediate or final vertices take $\pi$-estimates from their direct predecessors.

According to the works [1, 6], $\lambda$-estimates are given as follows: if $A \in V$ is the initial or intermediate vertices which reflect $m$ events and $B$ is the direct successor of $A$ vertex, which has $n$-events, than for $1 \leq i \leq m$:

$$
\lambda_B(a_i) = \sum_{j=1}^{n} p(b_j/a_i) \cdot \lambda(b_j),
$$

where $p(b_j/a_i)$ is a conditional probability of $b_j$ event implementation under condition that event $a_i$ will be realized.

$\pi$-value is determined as follows: if $B$ is an intermediate or terminal vertex of the network, which reflects the $n$-events and $A$ vertex is a direct predecessor of the $B$ vertex, which reflects $m$ events than for $1 \leq i \leq n$:

$$
\pi_B(b_i) = \sum_{i=1}^{m} p(b_i/a_i) \cdot \pi_B(a_i),
$$

where $p(b_i/a_i)$ is the conditional probability of $a_i$ event under condition that event $b_i$ has happened.

Taking into account the obtained $\lambda$ and $\pi$ – estimates and values, we can get the summary conditional probabilities $P'$ in the vertices of network where the events have happened. For example, if the $a_j$ event has happened in the vertex $A$ of network, than $P'_{(a_j)} = \alpha \ast \lambda(a_j) \ast \pi(a_j)$, where $\alpha$ is a normalization constant used when the summary of the calculated $P'_{(a_j)}$ is not equal to 1. More detailed information on the assurance networks is carried out in the works [1, 6]. Let consider an example of the construction and calculation of the assurance network for the probabilistic estimation of the innovative product.

![Diagram](attachment:figure1.png)
Unconditional probabilities for both A and B vertices are as follows:

\[ A: p(a_1) = 0.55; p(a_2) = 0.45 \]  
\[ B: p(b_1) = 0.4; p(b_2) = 0.6 \]  

Unconditional probabilities for both C and D vertices are as follows:

\[ C: p(c_1) = 0.65; p(c_2) = 0.35 \]  
\[ D: p(d_1) = 0.3; p(d_2) = 0.7 \]  

For E, F and G vertices the unconditional probabilities with regard to the written unconditional probabilities in vertices A, B, C, D have the following values:

\[ p(e_1/a_1, b_1) = 0.85; p(e_2/a_1, b_2) = 0.15 \]  
\[ E: p(e_1/a_1, b_2) = 0.35; p(e_2/a_1, b_2) = 0.65 \]  
\[ p\left(\frac{e_1}{a_2, b_1}\right) = 0.95; p\left(\frac{e_2}{a_2, b_1}\right) = 0.05 \]

\[ p(f_1/c_1, d_1) = 0.90; p(f_2/c_1, d_1) = 0.10 \]  
\[ F: p(f_1/c_1, d_2) = 0.30; p(f_2/c_1, d_2) = 0.70 \]  
\[ p(f_1/c_2, d_1) = 0.25; p(f_2/c_2, d_1) = 0.75 \]  
\[ p(f_1/c_2, d_2) = 0.15; p(f_2/c_2, d_2) = 0.85 \]  
\[ p(g_1/e_1, f_1) = 0.90; p(g_2/e_1, f_1) = 0.10 \]  
\[ G: p(g_1/e_1, f_2) = 0.65; p(g_2/e_1, f_2) = 0.35 \]  
\[ p(g_1/e_2, f_1) = 0.45; p(g_2/e_2, f_1) = 0.55 \]  
\[ p(g_1/e_2, f_2) = 0.15; p(g_2/e_2, f_2) = 0.85 \]

In accordance with the algorithm [4] for the ranges of all \( \lambda \)-values, \( \lambda \)-estimates and \( \pi \)-estimates the unit values are set as equal to 1:

\[ \lambda(A) = (1;1); \lambda(B) = (1;1); \lambda(C) = (1;1); \lambda(E) = (1;1); \lambda(F) = (1;1); \lambda(G) = (1;1); \lambda(C) = (1;1); \lambda(D) = (1;1); \]

\[ \pi_E(A) = (1;1); \pi_E(B) = (1;1); \pi_E(C) = (1;1); \pi_E(D) = (1;1); \]

Since, the A, B, C, D vertices are the initial ones, than \( \pi \)-values are equal to the prior unconditional probability and they are established for these vertices:

\[ \pi(a_1) = 0.55; \pi(a_2) = 0.45; \pi(b_1) = 0.6; \pi(c_1) = 0.65; \pi(c_2) = 0.35; \]

\[ \pi(d_1) = 0.3; \pi(d_2) = 0.7. \]

E and F vertices are the direct successor of the initial vertices, so we have as follows:

\[ \pi_E(a_1) = p(a_1)/\lambda_E(a_1) = 0.55; \pi_E(a_2) = p(a_2)/\lambda_E(a_2) = 0.45; \]

\[ \pi_E(b_1) = p(b_1)/\lambda_E(b_1) = 0.4; \pi_E(b_2) = p(b_2)/\lambda_E(b_2) = 0.6; \]

\[ \pi_F(c_1) = p(c_1)/\lambda_F(c_1) = 0.65; \]

\[ \pi_F(d_1) = p(d_1)/\lambda_F(d_1) = 0.3; \pi_F(d_2) = p(d_2)/\lambda_F(d_2) = 0.7. \]

Let calculate the vectors of the full conditional probabilities \( P(E) \) and \( P(F) \) of the occurrences in the E and F vertices taking into account all previous information. The calculation for the E vertex is performed as follows:

\[ \pi(e_1) = p(e_1/a_1,b_1) \ast \pi_E(a_1) \ast \pi_E(b_1) \]

\[ + p(e_1/a_2,b_2) \ast \pi_E(a_2) \ast \pi_E(b_1) + p(e_1/a_2,b_2) \ast \pi_E(a_2) \ast \pi_E(b_2) \]

\[ = 0.85 \ast 0.55 \ast 0.4 + 0.35 \ast 0.55 \ast 0.6 + 0.45 \ast 0.4 \ast 0.95 + 0.45 \ast 0.5 \ast 0.6 = 0.443. \]

\[ \pi(e_2) = p(e_2/a_1,b_1) \ast \pi_E(a_1) \ast \pi_E(b_1) \]

\[ + p(e_2/a_2,b_2) \ast \pi_E(a_2) \ast \pi_E(b_2) \]

\[ = 0.15 \ast 0.55 \ast 0.4 + 0.65 \ast 0.4 \ast 0.55 + 0.05 \ast 0.6 \ast 0.95 \]

\[ + 0.05 \ast 0.4 \ast 0.6 = 0.405. \]

\[ P'(e_1) = \alpha \ast \lambda(e_1) \ast \pi(e_1) = \alpha \ast 1 \ast 0.443 = 0.443 \alpha; \]

\[ P'(e_2) = \alpha \ast \lambda(e_2) \ast \pi(e_2) = \alpha \ast 1 \ast 0.405 = 0.405 \alpha; \]

Where \( \alpha \) is a normalization constant.

We finally obtain:

\[ P'(e_1) = 0.522 \]

\[ P'(e_2) = 0.478 \]

The calculations for the vertex F is performed as follows:
\[ \pi(f_1) = p(f_1/c_1 d_1) \ast \pi_F(c_1) \ast \pi_F(d_1) + p(f_1/c_2 d_2) \ast \pi_F(c_2) \ast \pi_F(d_2) + p(f_1/c_3 d_3) \ast \pi_F(c_3) \ast \pi_F(d_3) = 0.90 \ast 0.65 \ast 0.3 + 0.3 \ast 0.65 \ast 0.7 + 0.25 \ast 0.35 \ast 0.3 + 0.15 \ast 0.35 \ast 0.7 = 0.375; \]
\[ \pi(f_2) = p(f_2/c_1 d_1) \ast \pi_F(c_1) \ast \pi_F(d_1) + p(f_2/c_2 d_2) \ast \pi_F(c_2) \ast \pi_F(d_2) + p(f_2/c_3 d_3) \ast \pi_F(c_3) \ast \pi_F(d_3) = 0.1 \ast 0.65 \ast 0.3 + 0.7 \ast 0.65 \ast 0.7 + 0.75 \ast 0.35 \ast 0.3 + 0.85 \ast 0.35 \ast 0.7 = 0.554. \]

After use of the normalization constant, we finally obtain:
\[ P'(f_1) = 0.35 \ast \lambda(f_1) \ast \pi(f_1) = 0.35 \ast 1 \ast 0.375 = 0.375 \alpha; \]
\[ P'(f_2) = 0.65 \ast \lambda(f_2) \ast \pi(f_2) = 0.65 \ast 1 \ast 0.405 = 0.554 \alpha. \]

We then have for the vertex G:
\[ \pi(g_1) = p(g_1/e_1 f_1) \ast \pi_G(e_1) \ast \pi_G(f_1) + p(g_1/e_2 f_2) \ast \pi_G(e_2) \ast \pi_G(f_2) + p(g_1/e_3 f_3) \ast \pi_G(e_3) \ast \pi_G(f_3) = 0.9 \ast 0.522 \ast 0.403 + 0.65 \ast 0.522 \ast 0.597 + 0.45 \ast 0.478 \ast 0.403 \ast 0.15 \ast 0.478 \ast 0.597 = 0.520; \]
\[ \pi(g_2) = p(g_2/e_1 f_1) \ast \pi_G(e_1) \ast \pi_G(f_1) + p(g_2/e_2 f_2) \ast \pi_G(e_2) \ast \pi_G(f_2) + p(g_2/e_3 f_3) \ast \pi_G(e_3) \ast \pi_G(f_3) = 0.1 \ast 0.522 \ast 0.403 + 0.35 \ast 0.522 \ast 0.597 + 0.55 \ast 0.478 \ast 0.403 + 0.85 \ast 0.478 \ast 0.597 = 0.480. \]

For the obtained values we don’t need to multiply the expression by the normalization constant, so we finally have:
\[ P'(g_1) = \lambda(g_1) \ast \pi(g_1) = 1 \ast 0.520 = 0.520; \]
\[ P'(g_2) = \lambda(g_2) \ast \pi(g_2) = 1 \ast 0.480 = 0.480. \]

On the base of the obtained results of probability propagation along the network we conclude that the probability of the satisfactory state of portfolio of innovative product is very high \((P'(g_1) = 0.520)\). This is insured by the significant probabilities of innovative level of the product \((P'(f_2) = 0.554)\) and sales volume \(P'(f_2) = 0.554\). Such situation may arise when the new product enters the market for the first time ever.

Let assume that the \(a\)-event has happened in the vertex \(A\). The event points out that the level of the market competition is high so it is assessed that the probabilities in the vertex \(A\) shall be overestimated:
\[ P'(a_1) = 1; P'(a_2) = 0; \lambda(a_1) = 1; \lambda(a_2) = 0; \]
\[ \pi_E(a_1) = 1; \pi_E(a_2) = 0. \]

For the vertex \(E\) the estimations taken from the vertex \(B\) will have the same values but new \(\pi\)-value taken from the vertex \(A\) will be obtained.

We have for the vertex \(E\) as follows:
\[ \pi(e_1) = p(e_1/a_1 b_1) \ast \pi_E(a_1) \ast \pi_E(b_1) + p(e_1/a_2 b_2) \ast \pi_E(a_2) \ast \pi_E(b_2) + p(e_1/a_3 b_3) \ast \pi_E(a_3) \ast \pi_E(b_3) \]
\[ \pi(e_2) = p(e_2/a_1 b_1) \ast \pi_E(a_1) \ast \pi_E(b_1) + p(e_2/a_2 b_2) \ast \pi_E(a_2) \ast \pi_E(b_2) + p(e_2/a_3 b_3) \ast \pi_E(a_3) \ast \pi_E(b_3) = 1.5 \ast 0.4 \ast 0.6 + 0.05 \ast 0 \ast 0.4 \ast 0.05 + 0 \ast 0 \ast 0.6 = 0.45; \]
\[ P'(e_1) = P'(e_1/a_1) = 0.55; \]
\[ P'(e_2) = P'(e_2/a_1) = 0.45. \]

In the network arm \((C, D) \rightarrow F \rightarrow G\) there are not any changes in view of occurrence of the event, so in this arm the previously obtained calculated priori probabilities are used.

For the vertex \(G\) the calculations are performed as follows:
\[ \pi(g_1) = p(g_1/e_1 f_1) \ast \pi_G(e_1) \ast \pi_G(f_1) \ast \pi_G(g_1) + p(g_1/e_2 f_2) \ast \pi_G(e_2) \ast \pi_G(f_2) \ast \pi_G(g_1) + p(g_1/e_3 f_3) \ast \pi_G(e_3) \ast \pi_G(f_3) \ast \pi_G(g_1) = 0.9 \ast 0.55 \ast 0.403 \ast 0.65 \ast 0.55 \ast 0.597 \ast 0.45 = 0.45 \ast 0.403 \ast 0.15 \ast 0.45 \ast 0.597 = 0.54; \]
\[ \pi(g_2) = p(g_2/e_1 f_1) \ast \pi_G(e_1) \ast \pi_G(f_1) \ast \pi_G(g_2) + p(g_2/e_2 f_2) \ast \pi_G(e_2) \ast \pi_G(f_2) \ast \pi_G(g_2) + p(g_2/e_3 f_3) \ast \pi_G(e_3) \ast \pi_G(f_3) \ast \pi_G(g_2) = 0.1 \ast 0.55 \ast 0.403 \ast 0.35 \ast 0.55 \ast 0.597 \ast 0.55 \ast 0.45 \ast 0.403 \ast 0.85 \ast 0.45 \ast 0.597 = 0.46. \]

The obtained results differ slightly from the results of the network calculation before occurrence of the event \(a_1\). It can be considered as a confirmation of the initial inferences.

**Conclusions**

In this paper we discuss the example of modeling risk-contributing factors introducing various uncertainties in the innovation activities of enterprises on the base of probabilistic inference. For this purpose we use the assurances network, this method in contrast to, for example, the probability tree, allows to model complex uncertain situations with a large number of casual events and to take into account the stochastic relations between them. The main difficulty in implementation of the assurances networks is a large amount of calculations which increases dramatically with the number of network vertices. However, the use of appropriate software products helps to eliminate shortcomings in many instances.
References

