DOI: 10.32347/2412-9933.2024.57.102-106
UDC 539.3

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THE INFLUENCE OF GEOMETRICAL SHAPE ON THE BUCKLING OF THIN-WALLED AXISYMMETRIC SHELLS

Abstract. *The most economical for long span structures are thin-walled covering consisting of shell elements. These include shells of cylindrical, spherical, conical and other shapes. Buckling analysis of thin-walled structures is important in the design of buildings and structures. In this case, the geometric shape of the shell used has a great effect on the critical load. A comparative analysis of the geometrically nonlinear deformation, buckling, and postbuckling behavior of rotation panels of the same volume under static thermomechanical loading is carried out. Spherical and conical shells having the same thickness, rise and weight are considered. Preheated shells are loaded with uniform external pressure. The calculation method is based on geometrically nonlinear relations of the three-dimensional theory of thermoelasticity without the use of simplifying hypotheses of shell theory and the use of a moment finite element scheme. A universal 3D isoparametric finite element is used. The element allows you to model shells of stepwise variable thickness, with breaks in the middle surface and with other geometric features in thickness. The problem of nonlinear deformation, buckling, and postbuckling behavior of inhomogeneous shells is solved by a combined algorithm that employs the parameter continuation method, a modified Newton–Kantorovich method, and a procedure for automatic correction of algorithm parameters. The method has been justified numerically in the authors' articles. Research has revealed the behavioral features of the compared shallow panels.*

Keywords: thin shallow shell; 3D finite element; geometrically nonlinear deformation; buckling; postbuckling behavior; moment finite-element scheme; thermomechanical load; comparative analysis

Introduction

Currently, the problems of analyzing the stress-strain state and buckling of flexible shells attract great attention. It is in the process of loss of stability and subsequent deformation of shells that their load-bearing capacity is usually exhausted. At the same time, the degree of this influence significantly depends on the type of load. A special place is occupied by the problems of static stability of shells exposed to thermomechanical effects. The geometric shape of the shell under consideration has a great influence on the critical load and the buckling shape. Due to the simplicity of their shape, spherical and conical panels are widely used as the ceiling of large-span structures. Conical shells are often used as fairing elements. Such shells may be subject to thermomechanical loads during operation.

The purpose of this study is to explore and compare the effect of the geometric shape of the shell and the magnitude of preheating on geometrically nonlinear deformation, buckling and postbuckling behavior of the structure. Spherical and conical panels having the same rise, thickness and weight are considered. The results obtained for these shells are compared with solutions obtained using the LIRA-SAPR software (SW).

Formulation of the problem

The method of solving static problems of nonlinear deformation, buckling, and postbuckling behavior of thin elastic inhomogeneous shells is based on the geometrically nonlinear equations of the 3D thermoelasticity theory and use of the moment finite-element scheme (MFES) [1 – 4]. A model of a linearly elastic continuous medium is used, the properties of which correspond to the generalized Duhamel–Neumann law. Large displacements with small deformations are assumed. A solid finite element (FE) with additional variable parameters has been developed. A unified calculation model based on the universal FE has been created. The model takes into account the multilayer structure of the material and the geometric features of the structural elements of the inhomogeneous shell: casing of varying thickness, ribs, cover plates, cavities, channels, holes, and sharp bends of the mid-surface. The problem of geometrically nonlinear deformation, buckling, and postbuckling behavior of inhomogeneous shells is solved by a combined algorithm that employs the parameter continuation method, a modified Newton–Kantorovich method, and a procedure for automatic correction of algorithm parameters.

Results and discussions

Research is carried out for spherical and conical panels with the rise $k = H/h = 3$ [5; 6]. Initial data: thickness $h = 0.01$ m, radius of support boundary $a = 100h$; elastic modulus $E = 19.6\cdot10^4$ MPa, Poisson's ratio $\nu = 0.3$, coefficient of linear thermal expansion $\alpha = 0.125\cdot10^{-4}$ deg $^{-1}$. The shells are clamped at the edges. The effect of the thermomechanical load on the panels consists of two stages: (i) the stress–strain state of the shell is perturbed by the temperature field and (ii) the panel is subjected to pressure, the temperature field remaining constant. The shells are heated by $T = 20^\circ C$ that corresponds to the dimensionless parameter of the temperature $\bar{t} = 2\alpha T(a/h)^2 = 5$. The results are presented in dimensionless form: $\bar{q} = a^4 q/(Eh^4)$, $\bar{u}^{1'} = u^{1'}/h$.

A comparative analysis of the obtained results has been carried out with the solutions of Kantor [7] and obtained using the LIRA-SAPR software [8]. Kantor solved this problem by the variational method in high approximations using the nonlinear theory of shallow shells.

The results obtained by using the MFES agree well with those on the entire "load–deflection" ("$\bar{q} - \bar{u}^{1'}$") curves during preheating and in the precritical domain (Fig. 1, Fig. 2), with insignificant discrepancy for the upper critical load $\bar{q}_{cr}^{up}$. The latter effect is due to the different accuracy of modeling the heating process, as well as the fact that the MFES uses spatial FEs, in contrast to the LIRA-SAPR SW, which uses flat shell FEs.

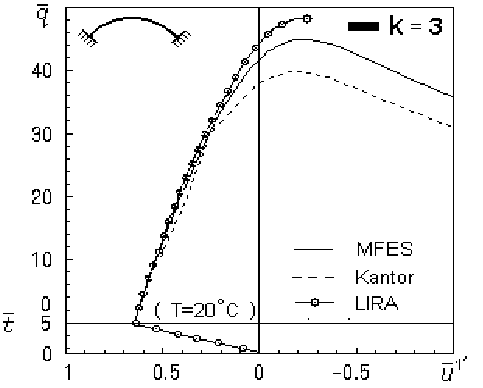


Figure 1 – The "$\bar{q} - \bar{u}^{1'}$" curves for preheated spherical panel

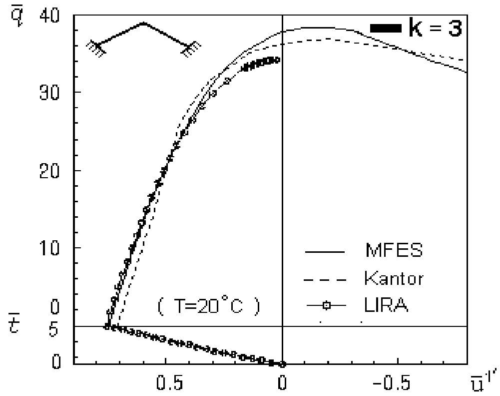


Figure 2 – The "$\bar{q} - \bar{u}^{1'}$" curves for preheated conical panel

A comparison of the influence of the shell shape on the nature of the "$\bar{q} - \bar{u}^{1'}$" curves is given in Fig. 3. To assess the effect of heating on the buckling of shells, corresponding solutions are shown for panels loaded only by pressure ($T = 0^\circ$ C, dash-dotted lines).

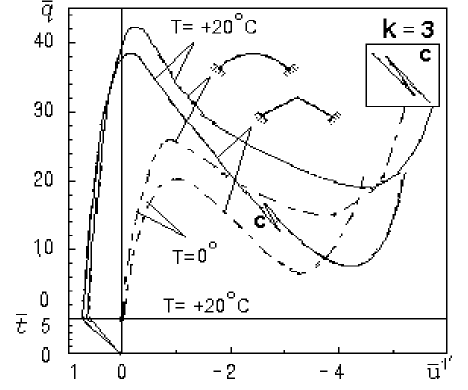


Figure 3 – Effect of geometric shape on the buckling of unheated and preheated panels

Comparing the behavior of spherical and conical shells, the following conclusions can be drawn. The volumes of the panels are almost the same; the difference is only 0.04%. However, changing the shape of the shell from a smooth spherical to a conical one with a singularity at the pole affects the stress-strain state of the structure. The heating causes deformation opposite to that induced by pressure. Thus, preheating significantly increases the stiffness of both panels, and the critical load $\bar{q}_{cr}^{up}$ increases by 1.65 and 1.89%, respectively, compared to unheated ($T = 0^\circ\mathrm{C}$) (Fig. 3). At the same time, the loads $\bar{q}_{cr}^{up}$ for the conical panel are less by 21.59% ($T = 0^\circ\mathrm{C}$) and 9.97% ($T = 20^\circ\mathrm{C}$) compared to those for the spherical shell.

The deformation modes of the panels at different stages of loading are shown in Fig. 4 and Fig. 5. The initial configuration of the shell is shown with a dash-dotted line.

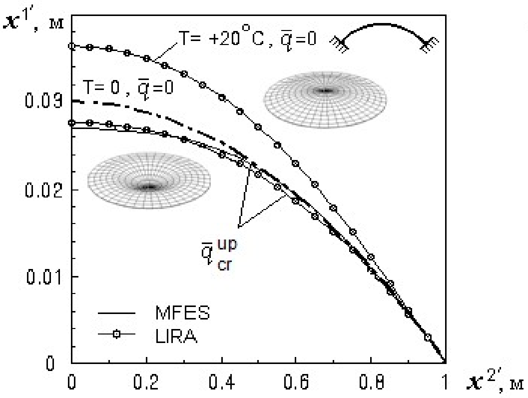


Figure 4 – Preheating shape of deformation and buckling shape of the spherical shell

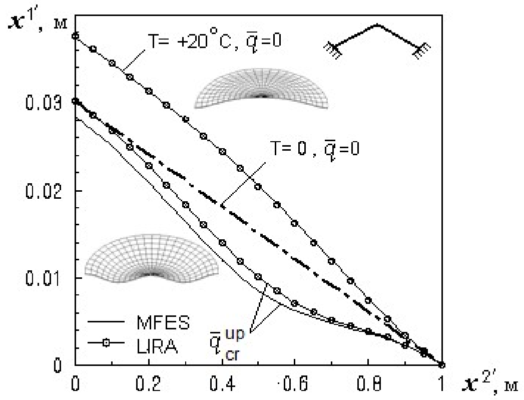
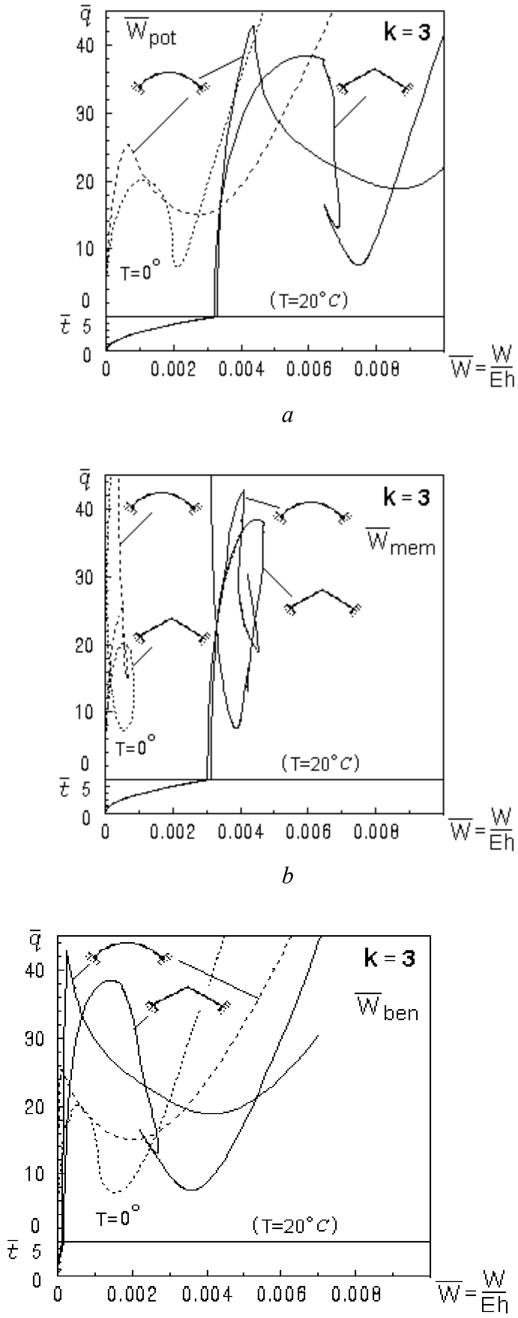


Figure 5 – Preheating shape of deformation and buckling shape of the conical shell

The action of the applied temperature field leads to a rise in the pole of the spherical panel by an amount $0.65h$, while the pole of the conical panel rises by an amount $0.8h$. The buckling form of the spherical panel is characterized by snapping at the pole, the conical shell loses stability with the formation of an axisymmetric dent in the middle of the meridian. The restructuring of the deformation shapes of the conical panel occurs at point "c" of the "$\bar{q} - \bar{u}^{1'}$" curve while maintaining the axisymmetric appearance.



a

b

c

Figure 6 – The "load-energy" curves for the spherical and conical shells

The "thermomechanical load – deformation energy" curves (Fig. 6) show the effect of preheating on the behavior of potential W_{pot} (a), membrane W_{mem} (b) and bending W_{ben} (c) energies. The dependence of energy on heating is nonlinear. The membrane component of energy predominates in the precritical domain. Preheating the shells significantly increases the level of deformation energy of the panels under pressure. The energy of the conical panel at the buckling moment is slightly greater than the energy of the spherical shell.

Conclusions

Investigations of the stability of spherical and conical shells of the same rise, thickness, and weight show that changing the geometric shape of the shell from a smooth spherical to a conical with a feature at the pole affects the stress-strain state and loss of stability of structures. The upper critical load for a spherical panel is greater than one for a conical shell. Buckling of the spherical panel occurs with snapping through at the center of the shell. Buckling of the conical shell is characterized by the formation of an axisymmetric dent in the middle of the meridian.

The presented solutions obtained using the moment finite element scheme are in good agreement with the solutions obtained using the LIRA-SAPR SW.

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The editorial board received the article on March 18, 2024

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ВПЛИВ ГЕОМЕТРИЧНОЇ ФОРМИ НА СТІЙКІСТЬ ТОНКОСТІННИХ ВІСЕСИМЕТРИЧНИХ ОБОЛОНОК

Анотація. *Найбільш економічним перекриттям для великопрогонових споруд є тонкостінне перекриття, що складається з оболонкових елементів. До них належать оболонки циліндричної, сферичної, конічної та інших форм. Аналіз стійкості конструкції є важливим елементом розрахунку під час проєктування будівель і споруд. При цьому великий вплив на критичне навантаження має геометрична форма оболонки, що використовується. Проведено порівняльний аналіз геометрично нелінійного деформування, стійкості та закритичної поведінки панелей однієї і тієї ж маси при статичному термомеханічному навантаженні. Розглянуто сферичну і конічну оболонки, що мають однакову товщину, висоту та масу. Попередньо нагріті оболонки навантажуються рівномірним зовнішнім тиском. Метод розрахунку заснований на геометрично нелінійних співвідношеннях тривимірної теорії термопружності без використання спрощуючих гіпотез теорії оболонок і використанні моментної схеми скінченних елементів.*

Застосовується універсальний просторовий ізопараметричний скінченний елемент. Елемент дає змогу моделювати оболонки ступінчасто-змінної товщини, зі зламами серединної поверхні та з іншими геометричними особливостями за товщиною. Задача нелінійного деформування, стійкості та закритичної поведінки неоднорідних оболонок розв'язується за допомогою комбінованого алгоритму, що використовує метод продовження розв'язку за параметром, модифікований метод Ньютона – Канторовича та процедуру автоматичного корегування параметрів алгоритму. Метод чисельно обґрунтовано в статтях авторів. Дослідження виявили особливості у поведінці порівнюваних пологих панелей.

Ключові слова: тонка полога оболонка; тривимірний скінченний елемент; геометрично нелінійне деформування; втрата стійкості; закритична поведінка; моментна скінченно-елементна схема; термомеханічне навантаження; порівняльний аналіз

Link to publication

APA Krivenko, Olga. P., Lizunov, Petro & Kalashnikov, Oleksandr. (2024). The influence of geometrical shape on the buckling of thin-walled axisymmetric shells. *Management of Development of Complex Systems*, 57, 102–106, dx.doi.org\10.32347/2412-9933.2024.57.102-106.

ДСТУ Кривенко О. П., Лізунов П. П., Калашніков О. Б. Вплив геометричної форми на стійкість тонкостінних вісесиметричних оболонок. *Управління розвитком складних систем.* Київ, 2024. № 57. С. 102 – 106, dx.doi.org\10.32347/2412-9933.2024.57.102-106.